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Consequences of an Abelian Family Symmetry

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ABSTRACT: The addition of an Abelian family symmetry to the Minimal Supersymmetric Standard Model reproduces the observed hierarchies of quark and lepton masses and quark mixing angles, only if it is anomalous. Green-Schwarz compensation of its anomalies requires the electroweak mixing angle to be $\sin^2 \theta_w = 3/8$ at the string scale, without any assumed GUT structure, suggesting a superstring origin for the standard model. The analysis is extended to neutrino masses and the lepton mixing matrix.

1. Introduction

The relative complexity of the standard model of strong and electroweak interactions suggests that it is the low energy manifestation of more fundamental theory. There are few hints as to the nature of this theory or value of the scale at which it becomes operative.

These questions can be studied in the context of the extension of the standard model to $N = 1$ supersymmetry[1] which allows for its perturbative extrapolation to near Planckian scales, where the gauge couplings [2]and some Yukawa couplings[3] appear to converge. This raises the hope that the $N = 1$ standard model at short distances is much simpler than at experimental scales, although we do not seem to have sufficient information to determine exactly the type of structure it describes, a GUT theory[4], or a direct descendant of superstrings. Fortunately there is more information, as not all parameters are the same at the unification scale, suggesting in fact a strong hierarchy among the masses of the quarks and the leptons, indicated by the orders of magnitude estimates[5]

$$\frac{m_u}{m_t} = \mathcal{O}(\lambda^8) \; ; \qquad \frac{m_c}{m_t} = \mathcal{O}(\lambda^4) \; ; \tag{1.1}$$

$$\frac{m_d}{m_b} = \mathcal{O}(\lambda^4) \; ; \qquad \frac{m_s}{m_b} = \mathcal{O}(\lambda^2) \; , \tag{1.2}$$

where, following Wolfenstein's parametrization[6], we use the Cabibbo angle λ, as expansion parameter. The charged lepton masses also satisfy similar relations

$$\frac{m_e}{m_\tau} = \mathcal{O}(\lambda^4) \; ; \qquad \frac{m_\mu}{m_\tau} = \mathcal{O}(\lambda^2) \; . \tag{1.3}$$

The mass hierarchy appears to be geometrical in each sector. The equality

$$m_b = m_\tau \; ,$$

known to be valid in the ultraviolet[3], yields the estimate

$$\frac{m_d m_s m_b}{m_e m_\mu m_\tau} = \mathcal{O}(1). \tag{1.4}$$

We assume that the mechanism which sets these orders of magnitude is an Abelian family symmetry, in the manner originally suggested by Froggatt and Nielsen[7].

In the following, we suggest that these orders of magnitude are determined by a gauged family Abelian symmetry. This is an old idea, some aspects of which has been revisited in the recent literature [8,9,10]. The work presented below is the result of a collaboration with P. Binétruy and S. Lavignac[11,12]. There are also closely related work in the recent literature, but with different emphases[13,14,15].

Our framework is the minimal extension of the Standard Model to $N = 1$ supersymmetry, including the so-called μ term, $P = \mu H_u H_d$. The most remarkable conclusion is that to reproduce these estimates, the symmetry must be anomalous. The anomalies can be compensated by the Green-Schwarz mechanism, fixing[16] $\sin^2 \theta_w = 3/8$, in perfect agreement with data when extrapolated to the infrared.

2. Quark Masses and Mixing Angles

The most general Abelian charge assignments to the particles of the Supersymmetric Standard Model can be written as

$$X = X_0 + X_3 + \sqrt{3}X_8 , \tag{2.1}$$

where X_0 is the family-independent part, X_3 is along λ_3, and X_8 is along λ_8, the two diagonal Gell-Mann matrices of the $SU(3)$ family space in each charge sector. In a basis where the entries correspond to the components in the family space of the fields $\mathbf{Q}$, $\overline{\mathbf{u}}$, $\overline{\mathbf{d}}$, L, and $\overline{e}$, we can write the different components in the form

$$X_i = (a_i, b_i, c_i, d_i, e_i) , \tag{2.2}$$

for $i = 0, 3, 8$. The Higgs doublets $H_{u,d}$ have zero X-charge because of the μ term.

The tree-level Yukawa coupling involves *only* the third family (implicitly choosing the third direction in family space),

$$y_t \mathbf{Q}_3 \overline{\mathbf{u}}_3 H_u + y_b \mathbf{Q}_3 \overline{\mathbf{d}}_3 H_d + y_\tau L_3 \overline{e}_3 H_d , \tag{2.3}$$

where the y_i's are the Yukawa couplings. This generates the relations

$$a_0 + b_0 = 2(a_8 + b_8) , \qquad a_0 + c_0 = 2(a_8 + c_8) , \qquad d_0 + e_0 = 2(d_8 + e_8) .$$

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The other elements of the Yukawa matrices are zero at tree-level because of X-charge conservation. Let x_{ij} be the excess X-charges at each of their entries; for the charge 2/3 Yukawa matrix they are

$$\begin{pmatrix} 3(a_8 + b_8) + a_3 + b_3 & 3(a_8 + b_8) + a_3 - b_3 & 3a_8 + a_3 \\ 3(a_8 + b_8) - a_3 + b_3 & 3(a_8 + b_8) - a_3 - b_3 & 3a_8 - a_3 \\ 3b_8 + b_3 & 3b_8 - b_3 & 0 \end{pmatrix} . \tag{2.4}$$

In the charge $-1/3$ sector, the b_i are replaced by the c_i, and in the charge -1 sector, a_i, b_i are replaced by d_i, e_i, respectively.

Introduce an electroweak singlet field θ with X-charge $-x$, to soak up the excess charge at each entry, x_{ij}, yielding an interaction of higher dimensions with no hypercharge [7,17]

$$\mathbf{Q}_i \overline{\mathbf{u}}_j H_u \left(\frac{\theta}{M_u} \right)^{n_{ij}} , \tag{2.5}$$

where the n_{ij} are positive numbers which satisfy

$$x_{ij} - x n_{ij} = 0 , \tag{2.6}$$

and M_u is some large scale. In a perturbative framework, the n_{ij} are expected to be integers. With only one field θ, not chaperoned by its vectorlike partner, invariance under supersymmetry then naturally[8] generates a true texture zero whenever a Yukawa matrix element has negative excess X-charge in units of $(-x)$, and non-zero entries correspond only to positive excess X-charge. Henceforth we normalize X so that $x = 1$.

We assume that the electroweak singlet θ develops a vacuum expectation value smaller than M_u, producing a *small parameter*, $\lambda_u \sim \theta/M_u$. This is what happens in many compactified superstring theories[18]. The n_{ij} then determine the order of magnitude of the entries in the Yukawa matrices[7]. It may seem that because the masses M_u, and thus the expansion parameters are in principle different in the three charge sectors, we are introducing many unknowns in our description, but most of the conclusions we can reach with this simple assumption depend only on the *existence* of these small parameters, *not*

on their values. Also, since the down quark and lepton sectors share the same electroweak quantum numbers, we expect them to be the same for the charge -1 and -1/3 matrices.

In the following we restrict ourselves to the case where all the excess charges in each Yukawa matrix are positive. We leave to future work the study of the cases where some of the excess charges are negative, which creates a true zero in that matrix element[8]. The charge 2/3 Yukawa matrix is

$$\mathbf{Y}_{uij} = \mathcal{O}(\lambda_u^{n_{ij}}) \ , \tag{2.7}$$

normalized to the top quark mass. It is not hard to diagonalize this matrix, setting

$$\mathbf{Y}_u = U_u D_u V_u^\dagger \ , \tag{2.8}$$

where

$$D_u = \text{diag} \ (\mathcal{O}(\lambda_u^{3(a_8+b_8)+a_3+b_3}), \mathcal{O}(\lambda_u^{3(a_8+b_8)-a_3-b_3}), \mathcal{O}(1)) \ ,$$

and the unitary matrix U_u is given by

$$U_u = \begin{pmatrix} \mathcal{O}(1) & \mathcal{O}(\lambda_u^{2a_3}) & \mathcal{O}(\lambda_u^{3a_8+a_3}) \\ \mathcal{O}(\lambda_u^{2a_3}) & \mathcal{O}(1) & \mathcal{O}(\lambda_u^{3a_8-a_3}) \\ \mathcal{O}(\lambda_u^{3a_8+a_3}) & \mathcal{O}(\lambda_u^{3a_8-a_3}) & \mathcal{O}(1) \end{pmatrix} \ , \tag{2.9}$$

These are valid for a range of parameters such that

$$3a_8 + 3b_8 > a_3 + b_3 > 0 \ .$$

We have a similar relation in the down quark sector, with the b_i replaced by c_i. It follows that the orders of magnitude of U_u and U_d are the same, but the expansion coefficients might be different. Let us set $\lambda_u = \lambda_d^y$, with $y > 0$. If $y > 1$, the orders of magnitude of the entries of the CKM matrix are

$$\mathcal{U}_{CKM} = \begin{pmatrix} \mathcal{O}(1) & \mathcal{O}(\lambda_d^{2a_3}) & \mathcal{O}(\lambda_d^{3a_8+a_3}) \\ \mathcal{O}(\lambda_d^{2a_3}) & \mathcal{O}(1) & \mathcal{O}(\lambda_d^{3a_8-a_3}) \\ \mathcal{O}(\lambda_d^{3a_8+a_3}) & \mathcal{O}(\lambda_d^{3a_8-a_3}) & \mathcal{O}(1) \end{pmatrix} . \tag{2.10}$$

If $y < 1$, the expansion parameter in the above is replaced by λ_u, that is its exponents all are multiplied by y. In either case the exponents satisfy the sum rule

$$n_{12} = n_{13} - n_{23} \ , \tag{2.11}$$

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which implies that

$$\frac{V_{us} V_{cb}}{V_{ub}} = \mathcal{O}(1) , \qquad (2.12)$$

in agreement with data (the right hand side is ≈ 3, and the Wolfenstein parametrization $(n_{12} = 1,\ n_{13} = 3,\ n_{23} = 2)$. We note the relation between our expansion parameters with the Cabibbo angle

$$\lambda \equiv V_{us} = \lambda_{u,d}^{2a_3} , \qquad (2.13)$$

depending on the relative magnitudes of λ_u and λ_d. The eigenvalue order of magnitude estimates are

$$\frac{m_u}{m_t} = \mathcal{O}(\lambda_u^{3(a_8+b_8)+a_3+b_3}) ; \qquad \frac{m_c}{m_t} = \mathcal{O}(\lambda_u^{3(a_8+b_8)-a_3-b_3}) ;$$

$$\frac{m_d}{m_b} = \mathcal{O}(\lambda_d^{3(a_8+c_8)+a_3+c_3}) ; \qquad \frac{m_s}{m_b} = \mathcal{O}(\lambda_d^{3(a_8+c_8)-a_3-c_3}) ,$$

The geometric hierarchy of the mass ratios in each quark sector suggests the further equalities

$$a_8 + b_8 = a_3 + b_3 ; \qquad a_8 + c_8 = a_3 + c_3 . \qquad (2.14)$$

Agreement with experimental information on the quark mass ratios dictates the following

$$2(a_8 + c_8) = y(a_8 + b_8) . \qquad (2.15)$$

In addition the mixing angle relation

$$V_{us} = \sqrt{\frac{m_d}{m_s}} , \qquad (2.16)$$

is satisfied provided that

$$c_3 = a_3 , \qquad (2.17)$$

if $y > 1$, and

$$c_3 = (2y - 1)a_3 , \qquad (2.18)$$

if $0 < y < 1$. We also find that

$$V_{cb} = \mathcal{O}(V_{us}^{\frac{3a_8-a_3}{2a_3}}) , \qquad (2.19)$$

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from which we may deduce that

$$3a_8 = 5a_3 \ . \tag{2.20}$$

Comparison with the data gives us six equations among seven unknown. The last unknown is y. Until we know the origins of the scales and of the expansion parameters, we cannot fix the values of λ_d and of λ_u in terms of observables. We note, for example, the interesting case $y = 2$, corresponding to $\lambda_u = \lambda_d^2$, yields $b_i = c_i$, which suggests an $SU(2)_R$ symmetry. It is quite remarkable that this simple idea is in agreement with the present data, and even predicts one successful relation among the CKM matrix elements (2.12).

3. Lepton Masses and Mixing Angles

An analysis akin to that in the previous section yields the charged lepton mass estimates

$$\frac{m_e}{m_\tau} = \mathcal{O}(\lambda_e^{3(d_8+e_8)+d_3+e_3}) \ ; \qquad \frac{m_\mu}{m_\tau} = \mathcal{O}(\lambda_e^{3(d_8+e_8)-d_3-e_3}) \ . \tag{3.1}$$

Geometric hierarchy of the charged lepton mass ratios implies that

$$d_8 + e_8 = d_3 + e_3 \ , \tag{3.2}$$

There are no mixing angles if the neutrinos are massless. Below, we generalize the Froggatt-Nielsen analysis to massive neutrinos, without assuming any extra symmetry[19]. We do this by adding right-handed neutrinos to the MSSM in order to generate masses for the neutrinos via the "see-saw" mechanism[20].

Let us assume that the low energy chiral remnants of the primal soup come from **27** representations of E_6. This representation carries two fields with no electroweak quantum numbers. One is an $SO(10)$ singlet, as we can see from the decomposition $\mathbf{27} = \mathbf{16} \oplus \mathbf{10} \oplus \mathbf{1}$. The other is an $SU(5)$ singlet which lives in the spinor representation of $SO(10)$ $\mathbf{16} = \mathbf{\bar{5}} \oplus \mathbf{10} \oplus \mathbf{1}$. This same field is part of an isodoublet under the right-handed $SU(2)_R$ inside $SO(10)$ $\mathbf{16} = (\mathbf{2}, \mathbf{1}, \mathbf{\bar{3}}^c \oplus \mathbf{1}^c) \oplus (\mathbf{1}, \mathbf{2}, \mathbf{3}^c \oplus \mathbf{1}^c)$. These two neutrino fields are not so "ino" as they are assumed to be very massive. With two fields, the Majorana mass matrix is

$$\begin{pmatrix} 0 & m_1 \ m_2 \\ \begin{matrix} m_1 \\ m_2 \end{matrix} & \mathcal{M}_0 \end{pmatrix} \ ,$$

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where $\mathcal{M}_0$ is a 2×2 symmetric matrix, and $m_{1,2}$ are the usual $\Delta I_w = 1/2$ mass entries of electroweak order. Let the eigenvalues of $\mathcal{M}_0$ be M_1 and M_2. We can go to a basis where M_0 is diagonal, in which $m_{1,2}$ are rotated into $\hat{m}_{1,2}$, yielding the light eigenvalue

$$m_\nu = \frac{\hat{m}_1^2 M_2 - \hat{m}_2^2 M_1}{M_1 M_2} \ .$$

Thus if $M_1 < M_2$, it becomes just $\hat{m}_1^2/M_1$, so that it is the lighter of the singlet neutrinos that enters in the light neutrino mass. This assumes that the $\hat{m}_i$ are of the same order of magnitude, themselves much smaller than M_1.

Thus in the following we assume only one right-handed neutrino per family, and leave the more complicated analysis to others. Assume that we have three such fields, $\overline{N}_i$, each carrying X-charge. The superpotential now contains the new interaction terms

$$L_i \overline{N}_j H_u \left(\frac{\theta}{m_\nu} \right)^{p_{ij}} + m_0 \overline{N}_i \overline{N}_j \left(\frac{\theta}{m_0} \right)^{q_{ij}} \ , \tag{3.3}$$

multiplied by couplings of order one, and where m_0 is some mass of the order of the GUT scale or string scale. In analogy with the quark and charged lepton sectors, we assume that $p_{33} = 0$, so that there is only the tree-level coupling for the third family. Call the X-charges of the right-handed neutrinos f_0, f_3, f_8, so that at tree-level

$$d_0 + f_0 = 2(d_8 + f_8) \ . \tag{3.4}$$

All Yukawa couplings satisfy conservation of X, relating q_{ij} and p_{ij} to the X-charges of the fields. For three families, the 6×6 Majorana mass matrix is of the form

$$\begin{pmatrix} 0 & \mathcal{M} \\ \mathcal{M}^T & \mathcal{M}_0 \end{pmatrix} \ .$$

In the above $\mathcal{M}$ is the $\Delta I_w = 1/2$ mass matrix with entries not larger than the electroweak breaking scale, and $\mathcal{M}_0$ is the unrestricted $\Delta I_w = 0$ mass matrix. Assuming that the order of magnitude of the $\Delta I_w = 0$ masses is much larger than the electroweak scale, we obtain the generalized "see-saw" mechanism.

The calculation of the light neutrinos masses and mixing angles proceeds in two steps. Let U_0 be the matrix which diagonalizes $\mathcal{M}_0$, that is

$$\mathcal{M}_0 = U_0 D_0 U_0^T , \tag{3.5}$$

where D_0 is diagonal. Then in terms of D_ν, the 3×3 eigenvalue matrix for the light neutrinos, and U_ν be their mixing matrix, we have

$$\mathbf{Y}'_e \equiv U_\nu D_\nu U_\nu^T = -\mathcal{M}' \frac{1}{D_0} \mathcal{M}'^T . \tag{3.6}$$

In the "see-saw" limit, the matrices U_0 and U_ν are unitary, so that

$$\mathcal{M}' = \mathcal{M} U_0^* . \tag{3.7}$$

The orders of magnitude of the heavy neutrino mass matrix are

$$\mathcal{M}_0 = m_0 \mathcal{O} \begin{pmatrix} \lambda_0^{2(f_0+f_3+f_8)} & \lambda_0^{2(f_0+f_8)} & \lambda_0^{2f_0+f_3-f_8} \\ \lambda_0^{2(f_0+f_8)} & \lambda_0^{2(f_0-f_3+f_8)} & \lambda_0^{2f_0-f_3-f_8} \\ \lambda_0^{2f_0+f_3-f_8} & \lambda_0^{2f_0-f_3-f_8} & \lambda_0^{2(f_0-2f_8)} \end{pmatrix} .$$

Its diagonalization yields the three eigenvalues

$$M_1 = m_0 \mathcal{O}(\lambda_0^{2(f_0+f_3+f_8)}) \;<\; M_2 = m_0 \mathcal{O}(\lambda_0^{2(f_0-f_3+f_8)}) \;<\; M_3 = m_0 \mathcal{O}(\lambda_0^{2(f_0-2f_8)}) .$$

We have assumed for simplicity that the charges satisfy the inequalities

$$f_0 > 2f_8 , \qquad 3f_8 > f_3 > 0 , \tag{3.8}$$

corresponding to $M_1 < M_2 < M_3$. The diagonalizing matrix is

$$U_0 = m_0 \mathcal{O} \begin{pmatrix} 1 & \lambda_0^{2f_3} & \lambda_0^{3f_8+f_3} \\ \lambda_0^{2f_3} & 1 & \lambda_0^{3f_8-f_3} \\ \lambda_0^{3f_8+f_3} & \lambda_0^{3f_8-f_3} & 1 \end{pmatrix} .$$

The electroweak breaking mass yields the matrix

$$\mathcal{M} = m \mathcal{O} \begin{pmatrix} \lambda_\nu^{3(d_8+f_8)+d_3+f_3} & \lambda_\nu^{3(d_8+f_8)+d_3-f_3} & \lambda_\nu^{3d_8+d_3} \\ \lambda_\nu^{3(d_8+f_8)-d_3+f_3} & \lambda_\nu^{3(d_8+f_8)-d_3-f_3} & \lambda_\nu^{3d_8-d_3} \\ \lambda_\nu^{3f_8+f_3} & \lambda_\nu^{3f_8-f_3} & 1 \end{pmatrix} , \tag{3.9}$$

where λ_ν is the expansion parameter, and m is a mass of electroweak breaking size. If we let $\lambda_0 = \lambda_\nu^z$, with $z > 0$. When $z \geq 1$, we find that

$$\mathbf{Y}'_e = \frac{m^2}{M_1} \mathcal{O}(\lambda_\nu^{6f_8+2f_3}) \mathcal{O} \begin{pmatrix} \lambda_\nu^{6d_8+2d_3} & \lambda_\nu^{6d_8} & \lambda_\nu^{3d_8+d_3} \\ \lambda_\nu^{6d_8} & \lambda_\nu^{6d_8-2d_3} & \lambda_\nu^{3d_8-d_3} \\ \lambda_\nu^{3d_8+d_3} & \lambda_\nu^{3d_8-d_3} & 1 \end{pmatrix} , \tag{3.10}$$

is the matrix whose eigenvalues yield the light neutrino masses, and their mixing angles. It is diagonalized by the unitary matrix

$$U_\nu = \mathcal{O} \begin{pmatrix} 1 & \lambda_\nu^{2d_3} & \lambda_\nu^{3d_8+d_3} \\ \lambda_\nu^{2d_3} & 1 & \lambda_\nu^{3d_8-d_3} \\ \lambda_\nu^{3d_8+d_3} & \lambda_\nu^{3d_8-d_3} & 1 \end{pmatrix} . \tag{3.11}$$

The light neutrino masses are then

$$m_{\nu_1} = \frac{m^2}{M_1} \mathcal{O}(\lambda_\nu^{2(3f_8+3d_8+f_3+d_3)}),$$

$$m_{\nu_2} = \frac{m^2}{M_1} \mathcal{O}(\lambda_\nu^{2(3f_8+3d_8+f_3-d_3)}) , \tag{3.12}$$

$$m_{\nu_3} = \frac{m^2}{M_1} \mathcal{O}(\lambda_\nu^{2(3f_8+f_3)}).$$

In order to obtain the mixing matrix which appears in the charged lepton current, we must fold this matrix with that which diagonalizes the charged lepton masses. If we let $\lambda_\nu = \lambda_e^w$, with $w > 1$, the result is

$$\mathcal{U}_\nu = \mathcal{O} \begin{pmatrix} 1 & \lambda_e^{2d_3} & \lambda_e^{3d_8+d_3} \\ \lambda_e^{2d_3} & 1 & \lambda_e^{3d_8-d_3} \\ \lambda_e^{3d_8+d_3} & \lambda_e^{3d_8-d_3} & 1 \end{pmatrix} .$$

When $0 < w < 1$, the matrix has the same form with λ_e replaced by λ_ν. It is similar to the CKM matrix. The mixing in the charged lepton current was first proposed by Maki, Nakagawa and Sakata in 1962, so we call it the MNS matrix[21]. We note that its elements satisfy

$$V_{e\nu_\mu} V_{\mu\nu_\tau} \sim V_{e\nu_\tau} . \tag{3.13}$$

It may be that $\lambda_e = \lambda_d$ and $\lambda_u = \lambda_\nu$, since they have the same quantum numbers, implying $w = y$. We also have the relations

$$\frac{m_{\nu_1}}{m_{\nu_2}} \approx (V_{e\nu_\mu})^w ; \qquad \frac{m_{\nu_2}}{m_{\nu_3}} \approx (V_{\mu\nu_\tau})^w, \tag{3.14}$$

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valid only when $w > 1$. When $0 < w < 1$ the exponents in these relations is one. In this analysis, the lepton mixing matrix has the same structure as the CKM matrix. We have assumed a simple set of inequalities among the charges, to provide an example of our method. When $0 < z < 1$, the forms of the neutrino masses are the same except that $f_{3,8}$ appear multiplied by z. The mixing matrix is unchanged.

Unlike quark masses and mixing, we have little solid experimental information on the values of these parameters. The most compelling evidence for neutrino masses and mixings come from the MSW interpretation of the deficit observed in various solar neutrino fluxes. In this picture, the electron neutrino mixes with another neutrino (assumed here to be the muon neutrino) with a mixing angle θ_{12} such that

$$m_{\nu_1}^2 - m_{\nu_2}^2 \approx 7 \times 10^{-6} \text{ eV}^2 \; ; \qquad \sin^2 2\theta_{12} \approx 5 \times 10^{-3} \; . \tag{3.15}$$

The other piece of evidence comes from the deficit of muon neutrinos in the collision of cosmic rays with the atmosphere. If taken at face value, these suggest that the muon neutrinos oscillate into another species of neutrinos, say τ neutrinos, with a mixing angle θ_{23}, and masses such that

$$m_{\nu_2}^2 - m_{\nu_3}^2 \approx 2 \times 10^{-2} \text{ eV}^2 \; ; \qquad \sin^2 2\theta_{23} \geq .5 \; . \tag{3.16}$$

Fitting the parameters coming from the solar neutrino data is rather easy, suggesting that

$$V_{e\nu_\mu} \sim \lambda_e^{2d_3} \sim \lambda^2 \; ,$$

together with $m_{\nu_2} \approx 1$ meV. However it is not so easy to understand the atmospheric neutrino data. These imply

$$V_{\mu\nu_\tau} \sim \lambda_e^{3d_8 - d_3} = \mathcal{O}(1) \; .$$

The relations (3.14) then suggest that w has to be large. For example the value $\theta_{23} \sim \frac{\pi}{9}$ for which $\sin^2 2\theta_{23} = .34$, yields $m_{\nu_2}/m_{\nu_3} \sim .01$, for $w = 4$. This gives $m_{\nu_3} \approx .1$ eV, which marginally reproduces the "data", and fixes the lightest neutrino mass to $m_{\nu_1} \approx 10^{-13}$ eV! The heaviest neutrino weighs one tenth of an eV, not enough to be of use for structure

formation. Perhaps there are more light neutrals, coming from the extra neutral leptons in each E_6 or from *end* T in string compactification.

Generically, though, it is difficult to understand mixing angles of order one, as suggested by the atmospheric neutrino data. The existence of only small mixing angles in the quark sectors suggests either that the interpretation of the atmospheric neutrino data is premature, or that there is fine tuning in the neutrino matrices[22].

4. Anomalies

The X family symmetry is in general anomalous. The three chiral families contribute to the mixed gauge anomalies as follows

$$C_3 = 3(2a_0 + b_0 + c_0) \ , \tag{4.1}$$

$$C_2 = 3(3a_0 + d_0) \ , \tag{4.2}$$

$$C_1 = a_0 + 8b_0 + 2c_0 + 3d_0 + 6e_0 \ . \tag{4.3}$$

The subscript denotes the gauge group of the Standard Model, *i.e.* $1 \sim U(1)$, $2 \sim SU(2)$, and $3 \sim SU(3)$. The X-charge also has a mixed gravitational anomaly, which is simply the trace of the X-charge,

$$C_g = 3(6a_0 + 3b_0 + 3c_0 + 2d_0 + e_0 + f_0) - x + C'_g, \tag{4.4}$$

where C'_g is the contribution from the particles that do not appear in the model we are discussing. One must also account for the mixed YXX anomaly, given by

$$C_{YXX} = 6(a_0^2 - 2b_0^2 + c_0^2 - d_0^2 + e_0^2) + 4A_T \ , \tag{4.5}$$

with the texture-dependent part given by

$$A_T = (3a_8^2 + a_3^2) - 2(3b_8^2 + b_3^2) + (3c_8^2 + c_3^2) - (3d_8^2 + d_3^2) + (3e_8^2 + e_3^2) \ . \tag{4.6}$$

The last anomaly coefficient is that of the X-charge itself, C_X, the sum of the cubes of the X-charge. Extra particles with chiral X-charge other than those in the minimal model, will contribute to both C'_g and C_X.

These anomaly coefficients can be related to combinations of quarks and lepton masses. The reason is that the X-charge of the determinant in each charge sector is *independent* of the texture coefficients that distinguish between the two lightest families. We set

$$\det \mathbf{Y}_u \sim y_t^3 \mathcal{O}(\lambda_u^U) , \qquad \det \mathbf{Y}_d \sim y_b^3 \mathcal{O}(\lambda_d^D) , \qquad \det \mathbf{Y}_l \sim y_\tau^3 \mathcal{O}(\lambda_e^E) , \qquad (4.7)$$

where

$$U \equiv 6(a_8 + b_8) , \quad D \equiv 6(a_8 + c_8) , \quad E \equiv 6(d_8 + e_8) .$$

Since the down and lepton matrices have the same quantum numbers, and couple to the same Higgs, we may assume they have the same expansion parameter, $\lambda_d = \lambda_e$. In that case we can relate the products of the down quark masses to that of the leptons (assuming $y_b = y_\tau$)

$$\frac{m_d m_s m_b}{m_e m_\mu m_\tau} \sim \mathcal{O}(\lambda_d^{(D-E)}) . \qquad (4.8)$$

From the tree-level Yukawa couplings to the third family expressed through (2.4), we can write combinations of anomaly coefficients in terms of the family-dependent charges

$$C_1 + C_2 - \frac{8}{3}C_3 = 12(d_8 + e_8 - a_8 - c_8) = 2(E - D) ,$$
$$C_3 = 6(2a_8 + b_8 + c_8) = U + D . \qquad (4.9)$$

These allow us to relate the anomaly coefficients to the ratio of products of quark and lepton masses (4.8), (assuming $y_b = y_\tau$),

$$\frac{m_d m_s m_b}{m_e m_\mu m_\tau} \sim \mathcal{O}(\lambda_d^{-(C_1+C_2-8/3C_3)/2}) . \qquad (4.10)$$

Compatibility with the extrapolated data requires the exponent to vanish

$$C_1 + C_2 - \frac{8}{3}C_3 = 0 , \qquad (4.11)$$

which expressed in other variables, reads $E = D$.

5. Green-Schwarz Cancellation of X Anomaly

If X is anomaly-free, then

$$C_1 = C_2 = C_3 = 0 , \qquad C_g = 0 . \qquad (5.1)$$

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The last equation is not constraining as there are likely more fields in the theory with chiral X-charge. These are consistent with (4.10), but the vanishing of C_3 contradicts our hypothesis that all excess charges have the same sign. Indeed, using the tree-level Yukawa relations (2.4), (4.1), we see that

$$0 = C_3 = 6(a_8 + b_8) + 6(a_8 + c_8) ,$$

which is not consistent with our assumption that all excess charges are positive. Hence we must rely on the Green-Schwarz mechanism.

String theories naturally contain all of the ingredients we need to reproduce the Yukawa textures. They have an antisymmetric tensor Kalb-Ramond field which in four dimensions is the Nambu-Goldstone boson of an anomalous $U(1)$ that couples like an axion through a dimension five term to the divergence of the anomalous current. Its anomalies are cancelled by the Green-Schwarz mechanism[23]. Under a chiral transformation, this term is capable of soaking up certain anomalies, by shifting the axion field, provided that they appear in commensurate ratios

$$\frac{C_i}{k_i} = \frac{C_X}{k_X} = \frac{C_g}{k_g} , \tag{5.2}$$

where the k_i are the Kac-Moody levels. They need to be integers only for the non-Abelian factors.

In superstring theories, this $U(1)$ is broken spontaneously slightly below the string scale. The scale is set by the charge content of the theory[18]. It follows that singlets with masses protected by X can still be very massive, and not appear in the effective low-energy theory.

This chiral U(1) X-charge can fix the value of the Weinberg angle, without the use of a grand unified group, as remarked by Ibàñez[16]. More recently, Ibàñez and Ross[9] applied it to the determination of symmetric textures when the field θ is vector-like.

In superstring theories, the non-Abelian gauge groups have the same Kac-Moody levels. For Green-Schwarz cancellation, it means that

$$C_2 = C_3 \qquad \text{or} \qquad d_0 = b_0 + c_0 - a_0 . \tag{5.3}$$

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After this very generic requirement, we see that equation (4.10) reduces to

$$\frac{m_d m_s m_b}{m_e m_\mu m_\tau} \sim \mathcal{O}(\lambda_d^{-(C_1 - 5/3 C_2)/2}) \ , \tag{5.4}$$

valid whenever θ is chiral. Since the right-hand side is of order one, it means that the exponent vanishes, so that in models with an *ab initio* μ term, we *deduce* that

$$C_1 = \frac{5}{3} C_2 \ . \tag{5.5}$$

However the gauge coupling constants at string unification scale with the anomaly coefficients, so that

$$\frac{C_1}{C_2} = \frac{g_1^2}{g_2^2} \ , \tag{5.6}$$

which fixes the Weinberg angle to the value

$$\sin^2 \theta_w = \frac{3}{8} \ ,$$

at the string scale, the canonical GUT value, but without the excess baggage of these theories! This is a strong hint that the $N = 1$ model does indeed come from superstrings!

We note that the mixed gravitational anomaly is exactly along the anomaly-free combination of baryon minus lepton numbers, $B - L$. In fact the most general X-charge can contain an arbitrary mixture along $B - L$, but this is already taken into account by our general parametrization. In superstring models, the Green-Schwarz mechanism extends to the mixed gravitational anomaly so that

$$\frac{C_g}{C_3} = \frac{k_g}{k_3} = \eta \ .$$

where η is a normalization parameter; in the simplest level-one models, it is equal to 12. In general, however,

$$C_g = \eta(U + E) \ . \tag{5.7}$$

The family independent X-charges are seen to depend only on two parameters, E, and U, assuming we know the normalization η.

Starting from very simple generic assumptions we are able to reproduce the data and even determine the Weinberg angle in terms of the ratio of quark and lepton masses. Our

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analysis, applied to neutrino masses, shows that it is ackward to accomodate both solar neutrino and atmospheric neutrino data.

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